10-6 Exponential Growth and Decay

Depreciation

$y = a(1 - r)^t$

Decay

$y = ae^{-kt}$

EX: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

$$65 = 130(1 - .11)^t$$

$$\frac{1}{2} = (.89)^t$$

$$\log \frac{1}{2} = t \log(.89)$$

$$\frac{\log(1/2)}{\log(.89)} = t = 5.95 \text{ hrs}$$

How long will it take for 90% of this caffeine to be eliminated from a person's body?

$$13 = 130(.89)^t$$

$$t = 19.76 \text{ hrs}$$

EX: The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years; that is, every 5760 years, half of a mass of Carbon-14 decays away.

What is the constant for Carbon-14?

$$y = ae^{-kt}$$

y - new amt
a - orignal
k - constant (rate)

$$\frac{1}{2} = 1e^{-k(5760)}$$

$$\frac{\ln \frac{1}{2}}{-5760} = \frac{-k\,5760}{-5760}$$

$$.00012 = k$$

$$1.2 \times 10^{-4}$$

A paleontologist examining the bones of a wooly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

$.03 = 1\, e^{-(.00012)t}$

$\ln .03 = -.00012t$

$29{,}221_{yrs} = t$

EX: The half-life of Sodium-22 is 2.6 years. What is the constant for Sodium-22?

$\frac{1}{2} = 1\, e^{-k\,2.6}$

$\frac{\ln \frac{1}{2}}{-2.6} = \frac{-2.6k}{-2.6}$

$.2666 = k$

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

$.1 = e^{-.2666t}$

$\ln .1 = -.2666t$

$8.6_{yrs} = t$

Appreciation

$y = a\,(1 + r)^t$

Growth

$y = ae^{kt}$

EX: In 1910 the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in the year 2010?

$$y = 120{,}000(1 + .015)^{100}$$

$$y = 531{,}845$$

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